

## Exercise 26

For the following exercises, use the Intermediate Value Theorem to confirm that the given polynomial has at least one zero within the given interval.

$$f(x) = x^5 - 2x, \text{ between } x = 1 \text{ and } x = 2.$$

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### Solution

Plug  $x = 1$  and  $x = 2$  into the function.

$$f(1) = (1)^5 - 2(1) = -1$$

$$f(2) = (2)^5 - 2(2) = 28$$

Since  $f(x)$  is a polynomial function (a smooth and continuous function),  $f(x)$  has to take on every value between  $-1$  and  $28$  for  $1 < x < 2$  by the Intermediate Value Theorem. Therefore,  $f(x)$  has a zero between  $x = 1$  and  $x = 2$ .